# Literature Review of Defeasible Entailment

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## ABSTRACT

Defeasible reasoning is a way to make conclusions in propositional logic that can handle exceptions to general rules. With this, we can make statements like "birds usually fly, but penguins do not fly", understand them and make further inferences. Defeasible entailment is a non-monotonic type of reasoning that allows for exceptions to general rules. It is useful for understanding situations where new information can change the conclusions made from a set of facts. This literature review looks at current research on defeasible entailment, including methods like Rational Closure and Lexicographic Closure in the KLM framework. We look at the strengths and weaknesses of these methods and identify areas for future research in this field.

## CCS CONCEPTS

• **Theory of computation → Automated reasoning**; • **Computing methodologies → Nonmonotonic, default reasoning and belief revision**.

## KEYWORDS

artificial intelligence, knowledge representation and reasoning, propositional logic, defeasible reasoning, defeasible entailment, rational closure, lexicographic closure

## 1 INTRODUCTION AND MOTIVATION

In simple terms, *Artificial Intelligence (AI)* is the development of machines capable of performing tasks that normally need human-like intelligence. The ability to derive information from existing knowledge occurs naturally for us humans. *"It was raining yesterday"* represents a simple natural language expression of an observation about one's environment whose meaning and related inferences are immediately clear to any reasonable human. Machines need more structured and rigorous ways to represent and manipulate information, no matter how simple.

*Knowledge Representation and Reasoning (KRR)* is an important field of research within AI with many practical applications and capabilities [14]. *KRR* seeks to address three primary concerns: how well to represent what we know about the world, how we deduce more or new knowledge from such a representation and, finally, how to manage the inverse relationship between knowledge articulateness (expressivity) and computational efficiency [7]. *Knowledge Representation* is the expression and maintenance of domain information about some environment using unambiguous language [11].

In knowledge-based systems, this domain of meaningful information is expressive and declarative and is syntactically encoded as a symbolic structure referred to as a *knowledge base* [12]. The application of a set of rules and manipulations to information in the knowledge base to derive more or new conclusions about the world describes the notion of *reasoning*.

*KRR* can be implemented using both logic-based and non-logic-based systems. Classical propositional logic is the simple but expressive language of representing knowledge [12]. Truth values are assigned to simple domain statements, known as *propositions*, and can be joined using operators to form more intricate logical statements [1]. Primarily, a logical statement or system is made up of two parts, its language composed of syntax and semantics plus a procedure of reasoning [11].

Defeasible entailment has the non-monotonicity property, hence it allows for previous inferences to be possibly withdrawn when new logically contradictory information is introduced. Unlike classical entailment, which has monotonicity properties, there is no one right way for defeasible entailment to work. As a result, there are many different ways to reason with defeasible knowledge. Kraus, Lehmann and Magidor [13] (KLM) proposed a set of properties that they believe define a good way to make conclusions with defeasible knowledge. This review will focus on the KLM framework because of their desirable theoretical and computational properties.

## 2 PROPOSITIONAL LOGIC

### 2.1 Syntax and Semantics

Propositional logic is a formal framework for representing human language information or knowledge and reasoning into logical statements [12]. Its simplicity and syntax make it a primary foundation for other logic systems, hence it is sometimes indicated as *zero-order logic*, and is the foundation for the rest of this paper. In propositional logic, the lowest form of a proposition is known as an *atom* and is assigned values of either **true** or **false** [1]. We can therefore attach meaning to each *atoms* with statements such as *'birds have wings'* and *'birds can fly'*.

The indivisible atoms in propositional logic are usually denoted using lowercase single letters such as $p$ and $q$. We can therefore define a finite set of all proportional atoms as $\mathcal{P} = \{p, q, \ldots\}$. We could represent the proposition *'birds have wings'* as $w$ and *'birds can fly'* as $f$. Propositional logic provides for the analysis of these statements without due regard to their instinctive meaning [12]. The figure below describes binary connectives, known as *Boolean operators*, which are employed recursively to join atoms or formulas to form other more complex formulas [5]:

| Name | Symbol | Example | Meaning |
|---|---|---|---|
| negation | $\neg$ | $\neg p$ | not $p$ |
| disjunction | $\vee$ | $p \vee q$ | $p$ or $q$ |
| conjunction | $\wedge$ | $p \wedge q$ | $p$ and $q$ |
| implication | $\rightarrow$ | $p \rightarrow q$ | $p$ if then $q$ |
| equivalence or bi-implication | $\leftrightarrow$ | $p \leftrightarrow q$ | $p$ if and only if $q$ |

**Figure 1: The Boolean operators used in propositional logic**

All the above connectives are binary, except ¬ which is unary, but they each accept all formulas as inputs [12]. Binary operators accept two operands while unary ones only take one operand. The *negation, disjunction and conjunction* boolean operators should be understood to mean and behave as the natural language words 'not', 'or, and 'and' respectively. As the natural language word 'implies' behaves, the *implication* connective means that, for example, if $\alpha = \mathcal{T}$ and $\alpha$ implies $\beta$, then $\beta = \mathcal{T}$ as well. As the visual symbol $\leftrightarrow$ explains *equivalence* is simply a two-way implication.

By convention, the boolean operators have assumed precedence in the following order, from highest to lowest: $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$. For an implication statement, or any of a similar format, such as $\alpha \rightarrow \beta$, we call $\alpha$ the *antecedent* and $\beta$ the *consequent* With these five base operators, we can form more complex propositional formulas such as $\neg(b \wedge w) \rightarrow \neg f$. The *proportional language* $\mathcal{L}$ is the set of all such *well-formed formulas* and its elements are denoted by lowercase Greek letters such as $\alpha, \beta, \theta, \ldots$. In *knowledge representation*, the finite set of formulas is referred to as a *knowledge base*, denoted by $\mathcal{K}$.

An *evaluation* or *interpretation* is an expression $\mathcal{P} \rightarrow \{T, F\}$, that designates a truth value to each entry, atom or formula, in $\mathcal{L}$. Iff a formula $\beta \in \mathcal{L}$ evaluates to **true** based on the interpretation of the truth values of statements and operator semantics in $\beta$, then we can state that $I \models \beta$, meaning $\beta$ is *satisfied* by the interpretation *I*. For instance, if we state that $I\ (b) = T$ and $I\ (w) = F$, then can evaluate that $I \models (b \vee w)$ and $I \models (b \wedge w)$. A *model of* $\beta$ is any evaluation that satisfied a formula $\beta$ and the set of all such models of $\beta$ is indicated as $Mod\ (\beta)$. Flowing from this understanding, we state that a formula fulfilled by every evaluation is denoted as $\top$ and one not fulfilled by any interpretation as $\bot$. For a knowledge base $\mathcal{K}$, the set of all interpretations is denoted by $\mathcal{U}$.

If a formula $\beta \in \mathcal{L}$ is made up of propositional atoms, $p$ and $q$, then we could assign a **true** (1) or **false** (0) value to these atoms individually through the values of the operands using truth tables. For easy interpretation, we evaluate a formula by breaking it down into its constituent *atoms* and then constructing a truth table based on the specific operators' semantics and precedence order. The resulting expression can be expressed as a sequence of atoms and barred atoms (e.g. $p, \overline{q}, r$) where the atom depicts the statement valuation being **true** and the barred one depicts a **false** statement valuation. The truth table below is an example of such a process.

| $p$ | $q$ | $\neg p$ | $\neg q$ | $p \vee q$ | $p \wedge q$ | $p \rightarrow q$ | $p \leftrightarrow q$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |

**Figure 2: A truth table for the Boolean operators $\neg, \wedge, \vee, \rightarrow$ and $\leftrightarrow$**

## 2.2 Limitations

The nature of classical proportional logic limits the ways in which information or knowledge can be communicated. This is because their monotonicity property dictates that the addition of any new information to the knowledge base, no matter how explicitly, does not remove or negate any prior inferences even when such seem to logically conflict with subsequent ones. [4] [17]. In other words, monotonicity allows for the expansion of the knowledge base and in turn, the drawing of new inferences but all existing inferences can never be retracted despite any new contradictory knowledge [12] [3]. This, therefore, poses a limiting challenge in how information could be adequately represented in the knowledge base and still draw conclusions that do not logically dispute the actual domain knowledge.

# 3 DEFEASIBLE REASONING

## 3.1 Motivation

The monotonicity property of entailment in classical propositional logic has caused many restrictive problems in the representation of knowledge in systems advanced by modern technology. As similarly described by Wang in [12], the example below describes a practical problem introduced by the monotonicity property:

*Example 3.1.* We have the following atoms representing the propositions "being a bird", "having wings", "able to fly", "being a penguin" and "being an ostrich" respectively:

$$\mathcal{P} = \{b, w, f, p, o\}$$

We can codify this information as:

- birds have wings ($b \rightarrow w$)
- birds can fly ($b \rightarrow f$)
- penguins are birds ($p \rightarrow b$)
- ostriches are birds ($o \rightarrow b$)

In proposition logic, we can represent this information as a *knowledge base* $\mathcal{K}$ with $\mathcal{K} = \{b \rightarrow w, b \rightarrow f, p \rightarrow b, o \rightarrow b\}$. We can reason logically and consequently infer using the principles of classical entailment that since penguins are birds, penguins can fly ($p \rightarrow f$). This deduction is correct and acceptable based on the current knowledge base. But we know that penguins can't fly ($p \rightarrow \neg f$), despite being birds. Our knowledge base $\mathcal{K}$ is now represented as $\mathcal{K} = \{b \rightarrow w, b \rightarrow f, p \rightarrow b, o \rightarrow b, p \rightarrow \neg f\}$.

As explained in the section 2.2, the monotonicity attribute of classical propositional logic does not negate our original conclusion that penguins can fly in spite of the explicit contradictory knowledge we just added [8]. Since there are no models of $\mathcal{K}$ in which $p$ can be true, we can only deduce that penguins don't exist and no further meaningful reasoning can be made about penguins.

We, therefore, need another reasonable form of entailment to handle cases such as the fact that penguins exit and are a non-flying class of birds. We can try to modify the knowledge base to categorically address penguins as being exceptional, but this avenue is not practical as other exceptions such as ostriches can't fly ($o \rightarrow \neg f$) would require similar structural changes to the knowledge base. $\not\approx$

Based on the *AI* principles of knowledge-based intelligent agents, our major interest is how to make logical actions based on some intrinsic representation of knowledge. The inability to convey the notion of *typicality* with classical proposition logic alone has been illustrated in example 3.1. Basically, we intended to communicate the concept that 'birds *typically* fly' and still have the proficiency to reason about the existence of penguins. The panacea is *nonmonotonicity* reasoning, which encompasses the tenets of "common sense" or *defeasible reasoning* that allows for the withdrawal of prior deductions when hostile additions are made to the knowledge base [12]. Many variations and approaches of defeasible reasoning have been advanced over the years but for our purpose, we will only consider the framework proposed by Kraus, Lehmann and Magidor (KLM), known as the *KLM Framework* [13]. The next section discusses what this entails and looks like.

## 4 DEFEASIBLE ENTAILMENT

Classical logic has limitations in conveying information due to its monotonicity property. This means that once something is inferred from a knowledge base, adding more information won't change it. As a result, previously inferred knowledge can't be retracted even when new statements are added.

KLM explains preferential entailment [13] and Lehmann and Magidor explain rank entailment [16] as versions of defeasible entailment. Both are still monotonic but furnish a foundation for defining nonmonotonic entailment based on preferential semantics. *Rational Closure* by Lehmann and Magidor [16] and *Lexicographic Closure* by Lehmann [15] are two such forms and are both LM-Rational compliant.

Rational Closure is more traditionalist or conservative and is categorised as a manifestation of prototypical reasoning while Lexicographic Closure is a form of presumptive reasoning. The fundamental distinction between these two classifications of reasoning is how much we can suppose based on the given knowledge or information. In prototypical reasoning, something is assumed true if it is true in the most 'typical' instances while in presumptive reasoning, something is assumed true if there is no contradictory proof.

It should be stated that other reasonable forms of defeasible entailment like *Relevant Closure* [2] are not LM-Rational, hence demonstrating that the KLM style is not the only valid alternative for stipulating defeasible entailment.

### 4.1 The KLM Framework

KLM contended that a nonmonotonic logic should be able to state vividly that "a $y$ is typically a $z$", and they explained that *"typically"* should be understood to mean "in a normal situation, it is reasonable to deduce $z$, given $y$". Guided by the expressivity limitations of classical logic for typicality communication, Lehmann and Magidor (LM) defined an avenue to record it by extending propositional logic [15][5].

Even though KLM defines other extensions, we will only concentrate on the initial *preferential consequence relation* over a propositional logic as explained by Kaliski [12]. With several postulates, this introduces a meta-level consequence relation denoted as $\mid\sim$ to represent *typicality* where the relation $\alpha \mid\sim \beta$, $\alpha$ and $\beta$ being propositional formulas, is read as "typically, if $\alpha$ then $\beta$. This review deals only with the KLM-style subclass of preferential reasoning (the core of nonmonotonic reasoning) known as *ranked interpretations* [15].

The following are the KLM properties for defeasible entailment:

(1) Reflexivity (Ref): $\mathcal{K} \models\!\approx \alpha \mid\sim \alpha$

(2) Left Logical Equivalence (LLE): $\dfrac{\alpha \equiv \beta,\ \mathcal{K} \models\!\approx \alpha \mid\sim \gamma}{\mathcal{K} \models\!\approx \beta \mid\sim \gamma}$

(3) Right Weakening (RW): $\dfrac{\mathcal{K} \models\!\approx \alpha \mid\sim \beta,\ \beta \models \gamma}{\mathcal{K} \models\!\approx \alpha \mid\sim \gamma}$

(4) And: $\dfrac{\mathcal{K} \models\!\approx \alpha \mid\sim \beta,\ \mathcal{K} \models\!\approx \alpha \mid\sim \gamma}{\mathcal{K} \models\!\approx \alpha \mid\sim \beta \wedge \gamma}$

(5) Or: $\dfrac{\mathcal{K} \models\!\approx \alpha \mid\sim \gamma,\ \mathcal{K} \models\!\approx \beta \mid\sim \gamma}{\mathcal{K} \models\!\approx \alpha \vee \beta \mid\sim \gamma}$

(6) Cautious Monotonicity (CM): $\dfrac{\mathcal{K} \models\!\approx \alpha \mid\sim \beta,\ \mathcal{K} \models\!\approx \alpha \mid\sim \gamma}{\mathcal{K} \models\!\approx \alpha \wedge \beta \mid\sim \gamma}$

(7) Rational Monotonicity (RM): $\dfrac{\mathcal{K} \models\!\approx \alpha \mid\sim \gamma,\ \mathcal{K} \not\models\!\approx \alpha \mid\sim \neg\beta}{\mathcal{K} \models\!\approx \alpha \wedge \beta \mid\sim \gamma}$

Any defeasible entailment that meets certain properties is known as *LM-Rational*. Property *(6)* above is included for consistency even though it is implied by the other properties. KLM explained the reasoning behind these properties when they were first defined [13]. Before discussing defeasible entailment further, we'll talk about preferential and ranked interpretations. These provide semantics for $\mid\sim$ and a foundation for defining LM-Rational forms of defeasible entailment.

#### 4.1.1 *Preferential Interpretations.*

Shoman [18] proposed that KLM's semantics for $\mid\sim$ are based on preferential logics. This expands classical logic semantics by introducing an ordering of interpretations based on *typicality*. A statement is considered true if it holds in the most typical interpretations. KLM initially defined defeasibility at the meta-level using a preferential consequence relationship and the first six KLM properties. They also defined a preferential interpretation consisting of a set of states, a mapping from states to propositional interpretations, and a strict partial order on the set of states. These preferential interpretations define preferential consequence relationships.

#### 4.1.2 *Ranked Interpretations.*

The preferential interpretations were refined by Lehmann and Magidor [16] through defining conditions that restrict the partial order to create ranked interpretations. This creates a series of non-empty levels where lower-level interpretations are more typical. The definition for a preferential consequence relation was also extended to define a rational consequence relation that satisfies all seven KLM properties. Ranked interpretations define rational consequence relations and every rational consequence relation can be defined by a ranked interpretation. Lehmann and Magidor also provided semantics for the operator $\mid\sim$ using ranked interpretations at the object level. A defeasible implication $\alpha \mid\sim \beta$ holds in a ranked interpretation $\mathcal{R}$ is true in all the most typical interpretations where $\alpha$ is true. It's worth noting that any propositional formula $\alpha$ can be written as an analogous defeasible implication $\alpha \mid\sim \top$ or $\neg\alpha \mid\sim \bot$ [5].

### 4.2 Rational Closure

The defeasible entailment relation $\models\!\approx_{RC}$ for Rational Closure is defined using the concept of *base ranks*. Described by Lehmann and Magidor [16] as a possible solution to the examination of what a defeasible knowledge base should entail, we can define Rational Closure using either a ranked interpretation or a ranking of declarations in the knowledge base $\mathcal{K}$. In the KLM framework, Rational Closure represents a prototypical pattern, one that is tremendously traditionalist in unusual instances of defeasible reasoning and is understood in terms of what postulations could follow from a given knowledge base $\mathcal{K}$. Lehmann and Magidor [16] further contended that other forms of entailment that are deemed reasonable even with more "adventurous" inferences should endorse those declarations in the Rational Closure of the corresponding knowledge base.

In the sections below, the definitions of Rational Closure will be discussed using ranked assertions followed by ranked formulas and the consequent algorithms it provides.

*4.2.1 Ranked Interpretations.* Lehmann and Magidor [16] explain that the minimal element corresponds to Rational Closure after the ordering on all rational consequence relations. Through the imposition of an ordering, $\preceq_{\mathcal{K}}$, on all ranked interpretations that are models of a knowledge base $\mathcal{K}$, Casini et al[5] also provides an associated explanation. We have $\mathcal{R}_1 \preceq_{\mathcal{K}} \mathcal{R}_2$ if $\mathcal{R}_1(u) \leq \mathcal{R}_2(u)$ for all $u \in \mathcal{U}$. This imposes another surface of typicality where lower-ranked interpretations are more typical. It is demonstrated by Giordano et al[10] that there is a distinct least model $\mathcal{R}^{RC}_{\mathcal{K}}$ for $\mathcal{K}$ where interpretations are thrust to the bottom as much as possible. A defeasible inference $\alpha \mid\sim \beta$ is in the Rational Closure of $\mathcal{K}$ if $\mathcal{R}^{RC}_{\mathcal{K}}$ is a model of $\alpha \mid\sim \beta$. Rational Closure is, therefore, LM-Rational because it is defined using ranked interpretations.

*4.2.2 Ranked Formulas.* Rational Closure can also be defined using the ranking of statements in a knowledge base $\mathcal{K}$ based on the base rank of individual formulas. The materialisation of a defeasible knowledge base $\mathcal{K}$, indicated as $\overrightarrow{\mathcal{K}}$, is where every defeasible implication is substituted by a classical implication [16] and represented formally as $\overrightarrow{\mathcal{K}} = \{\alpha \rightarrow \beta \mid \alpha \mid\sim \beta \in \mathcal{K}\}$.

With respect to $\mathcal{K}$, we considered a formula $\alpha$ exceptional if $\overrightarrow{\mathcal{K}} \models \alpha$, meaning it can be disproved using formulas in $\mathcal{K}$ [5]. Using exceptionality, we can define subsets of $\mathcal{K}$ using the function $\varepsilon(\mathcal{K}) = \{\alpha \mid\sim \beta\}$, $\alpha$ is exceptional with respect to $\mathcal{K}$. We set $\mathcal{E}^{\mathsf{K}}_0 = \mathcal{K}$ and successively set $\mathcal{E}^{\mathcal{K}}_{i+1} = \varepsilon(\mathcal{E}^{\mathcal{K}}_i)$ until we reach an $i$ where $\mathcal{E}^{\mathcal{K}}_{i+1} = \mathcal{E}^{\mathcal{K}}_i$. For this $i$ we set $\mathcal{E}^{\mathcal{K}}_\infty = \mathcal{E}^{\mathsf{K}}_i$. The base rank of a formula $\alpha$ (denoted as $br(\alpha)$ is then defined as the smallest $i$ such that $\alpha$ is not exceptional with respect to $\mathcal{E}^{\mathsf{K}}_i$.

$br(\alpha \mid\sim \beta)$ is defined as $br(\alpha)$ for any defeasible statement. For a knowledge base $\mathcal{K}$ and a defeasible formula $\alpha \mid\sim \beta$, the formula is in the Rational Closure of $\mathcal{K}$ if $br(\alpha) < br(\alpha \wedge \neg\beta)$ or $br(\alpha) = \infty$ [9].

*4.2.3 The Algorithms.* Rational Closure has an algorithm that ranks formulas in a knowledge base according to their specificity. More general statements have lower ranks. If there's an inconsistency, the most general information is discarded until it's resolved. Then classical entailment is used to compute entailment from the remaining knowledge base. This involves materializing the knowledge base and partitioning it into levels $\mathcal{R}_0, \ldots, \mathcal{R}_{n-1}, \mathcal{R}_\infty$ based on formula base ranks. The base rank of a formula is related to its defeasibility or typically, with lower-ranked formulas being more non-typical. This partitioning provides a specificity ranking of formulas. To inspect if $\alpha \mid\sim \beta$ is entailed by $\mathcal{K}$, we check if $\mathcal{R}_0 \cup \cdots \cup \mathcal{R}_{n-1} \cup \mathcal{R}_\infty \models \neg\alpha$. If this holds, we remove the lowest level $\mathcal{R}_0$ and inspect again until either only $\mathcal{R}_\infty$ remains or the stated entailment no longer holds. When this situation is reached, we evaluate $\mathcal{R}_i \cup \cdots \cup \mathcal{R}_{n-1} \cup \mathcal{R}_\infty \models \alpha \rightarrow \beta$ where $i$ is the level where it is returned as the result of the entailment, and when $\alpha$ is no longer exceptional. This algorithm makes it efficient to implement defeasible entailment with current classical reasoners by reducing it to a series of classical entailment checks.

## 4.3 Lexicographic Closure

Lehman [15] introduces Lexicographic Closure as a type of Defeasible Entailment. It can be defined using both a ranked interpretation and ranking formulas. The ranked formula definition is presented as an algorithm. Both definitions use a 'seriousness' ordering with two aspects: the number of violated formulas in an interpretation and the specificity of those violated formulas based on their base rank. The two criteria create two generate orderings that are fused in a lexicographic manner with specificity being the most important criterion.

*4.3.1 Ranked Interpretations.* To generate the ranked interpretation for Lexicographic Closure, interpretations are first ordered based on the specificity of the formulas they violate. Then, this ordering is refined using the number of violated formulas. It has been demonstrated by Casini et al[6] that Lexicographic Closure can be derived from Rational Closure, making it a refinement. The ranked interpretation for Lexicographic Closure can be obtained by taking the ranked interpretation for Rational Closure and ranking interpretations within levels based on the number of satisfied formulas. Interpretations that satisfy more formulas have a lower rank.

Formally, a ranking $\leq^{\mathcal{K}}_{LC}$ is defined where $u$, $u$, $v \in \mathcal{U}$, $u \leq^{\mathcal{K}}_{LC} u$ if $\mathcal{R}^{\mathcal{K}}_{RC}(u) = \infty$ or $\mathcal{R}^{\mathcal{K}}_{RC}(v) < \mathcal{R}^{\mathcal{K}}_{RC}(u)$ or $\mathcal{R}^{\mathcal{K}}_{RC}(v) = \mathcal{R}^{\mathcal{K}}_{RC}(u)$ and $C^{\mathcal{K}}(v) \leq C^{\mathcal{K}}(u)$, where $C^{\mathcal{K}}(v) = \{\alpha \mid\sim \beta \in \mathcal{K} \mid v \in Mod(\alpha \rightarrow \beta)\}$ and # indicates set size. A formula $\alpha \rightarrow \beta$ is in the Lexicographic Closure of a knowledge base  if the ranked interpretation obtained by ordering $\leq^{\mathcal{K}}_{LC}$ (denoted $\mathcal{R}^{\mathcal{K}}_{LC}$) is a model of $\alpha \mid\sim \beta$.

*4.3.2 Ranked Formulas.* A ranking of statements for Lexicographic Closure can also be obtained from Rational Closure and presented as an algorithm. The ranking for Rational Closure is refined by adding extra levels containing weakened versions of formulas from the knowledge base. Instead of removing an entire level of information when there's an inconsistency, the formula causing it is removed by weakening more general information. The algorithm ranks formulas in a knowledge base $\mathcal{K}$ according to the base rank algorithm in Rational Closure. When checking if $\mathcal{K}$ defeasibly entails a formula $\alpha \mid\sim \beta$, we start by checking if $\mathcal{R}_0 \cup \cdots \cup \mathcal{R}_{n-1} \cup \mathcal{R}_\infty \models \neg\alpha$.

If the entailment holds, instead of removing an entire level, the lowest level $\mathcal{R}_0$ is weakened. This is done by taking all subsets of size *x-1* (where *x* is the number of elements in the level), creating a single formula equivalent to all formulas in the subset using conjunction and then joining all combined formulas using disjunction to create a final formula. This considers all ways of removing a formula from $\mathcal{R}_0$. If $\alpha$ isn't exceptional with respect to the rest of the levels and this weakened version of $\mathcal{R}_0$, entailment is computed using classical entailment as before. Otherwise, $\mathcal{R}_0$ is weakened again considering subsets of size *x-2*. If zero size is reached for the subsets, $\mathcal{R}_0$ is discarded entirely and the process commences again with the next level. Like in Rational Closure where we did not discard the lowest level, we also don't weaken it in Lexicographic Closure.

The enfeeble levels can be computed and placed before computing entailment allowing the Lexicographic Closure algorithm to fit into Casini et al's [6] general pattern for defeasible entailment. However, this would result in unnecessary computation as higher levels often don't need their weakened versions added.

## 4.4 Limitations

Defeasible reasoning is useful for understanding situations where new information can change the conclusions made from a set of facts. However, it has its limitations. One limitation is that it can be hard to decide how careful to be when making conclusions. Some methods like Rational Closure are very careful and assume there are very few exceptions to general rules. Other methods like Lexicographic Closure are less careful and allow for conclusions even in unusual situations unless there is evidence against it. Deciding how careful to be can be difficult and depends on the situation and goals. Another limitation is that it can be hard to understand complex uncertainty [12][5].

Defeasible reasoning usually assumes new information supports or contradicts existing beliefs. But in real life, new information may only partly support or contradict existing beliefs. Understanding this type of uncertainty can be difficult. Lastly, defeasible reasoning can take a lot of time and effort. Many methods require looking through many possible conclusions to see which ones are supported by evidence. This takes time and may not work for quick decisions. In conclusion, while defeasible reasoning is useful for understanding uncertain situations, it has limitations. More research is needed to develop better methods that can handle complex uncertainty and take less time and effort [12] [7].

# 5 DISCUSSION

The examples for Rational Closure and Lexicographic Closure manifested their dissimilarities regarding their prototypical and presumptive reasoning techniques. However, this does not mean that Lexicographic Closure is necessarily a more valid form of defeasible entailment than Rational Closure just because it correctly entailed that penguins can not fly while Rational Closure did not. It is likely that there is no single correct form of defeasible entailment, but rather some forms may be better suited to certain domains than others.

Though not to an impactful extent, other approaches for defeasibility in logic have been explored. Casini et al[5] suggest using their method of rational closure in Description Logics by introducing defeasibility into *OWL Ontologies*. In trying to explain justifications for defeasible reason, Chama's [7] result for Rational Closure is promising because it suggests that similar justification algorithms may be produced for Relevant and Lexicographic Closure.

# 6 CONCLUSIONS

We demonstrated how formal logic can be used to model information and perform reasoning. By using entailment, we can infer new information and draw conclusions from knowledge bases. However, classical logic is too limiting, so we switched to nonmonotonic logic to reason with uncertainty. We examined defeasible reasoning in the KLM framework and discussed two approaches for defeasible entailment: Rational Closure and Lexicographic Closure.

We discussed the motivation and different applications and reasons for defeasible entailment. Rational Closure is a careful way to make conclusions in the KLM framework. It assumes that there are very few exceptions to general rules and makes conclusions based on this idea. However, other ways of making conclusions within this framework are less careful than Rational Closure. For example, Lexicographic Closure allows for conclusions even in unusual situations unless there is evidence against it.

Defeasible entailment is useful for understanding situations where new information can change the conclusions made from a set of facts. Methods like Rational Closure and Lexicographic Closure in the KLM framework provide good ways to represent and understand defeasible knowledge. However, there are still many questions to be answered and opportunities for future research in this area. More work is needed to develop better methods for defeasible entailment and to understand the strengths and weaknesses of current methods.

# REFERENCES


[1] Mordechai Ben-Ari. 2012. Propositional logic: Formulas, models, tableaux. *Mathematical Logic for Computer Science* (2012), 7–47.

[2] Giovanni Casini, Thomas Meyer, Kodylan Moodley, and Riku Nortjé. 2014. Relevant Closure: A New Form of Defeasible Reasoning for Description Logics. In *JELIA 2014: Logics in Artificial Intelligence*. Springer, Cham, Funchal, Madeira, Portugal, 92–106. https://doi.org/10.1007/978-3-319-11558-0_7

[3] Giovanni Casini, Thomas Meyer, Kody Moodley, Uli Sattler, and Ivan Varzinczak. 2015. Introducing defeasibility into OWL ontologies. In *The Semantic Web-ISWC 2015: 14th International Semantic Web Conference, Bethlehem, PA, USA, October 11-15, 2015, Proceedings, Part II 14*. Springer, 409–426.

[4] Giovanni Casini, Thomas Meyer, Kodylan Moodley, and Ivan Varzinczak. 2013. *Towards practical defeasible reasoning for description logics.* Technical Report. Centre for Artificial Intelligence Research. http://researchspace.csir.co.za/dspace/handle/10204/7039

[5] Giovanni Casini, Thomas Meyer, and Ivan Varzinczak. 2019. Taking Defeasible Entailment beyond Rational Closure. (2019), 18 pages.

[6] Giovanni Casini, Thomas Meyer, and Ivan Varzinczak. 2019. Taking Defeasible Entailment Beyond Rational Closure. In *Logics in Artificial Intelligence*, Francesco Calimeri, Nicola Leone, and Marco Manna (Eds.), Vol. 11468. Springer International Publishing, Cham, 182–197. https://doi.org/10.1007/978-3-030-19570-0_12

[7] Victoria Chama. 2020. *Explanation For Defeasible Entailment.* Master's thesis. Faculty of Science, University of Cape Town, Rondebosch, Cape Town, 7700.

[8] Dov M Gabbay. 1985. *Theoretical foundations for non-monotonic reasoning in expert systems.* Springer.

[9] L. Giordano, V. Gliozzi, N. Olivetti, and G.L. Pozzato. 2015. Semantic characterization of rational closure: From propositional logic to description logics. *Artificial Intelligence* 226 (sep 2015), 1–33. https://doi.org/10.1016/J.ARTINT.2015.05.001

[10] Laura Giordano, Valentina Gliozzi, Nicola Olivetti, and Gian Luca Pozzato. 2015. Semantic characterization of rational closure: From propositional logic to description logics. *Artificial Intelligence* 226 (2015), 1–33.

[11] Crina Grosan, Ajith Abraham, Crina Grosan, and Ajith Abraham. 2011. Knowledge representation and reasoning. *Intelligent Systems: A Modern Approach* (2011), 131–147.

[12] Adam Kaliski. 2020. *An Overview of KLM-Style Defeasible Entailment.* Master's thesis. Faculty of Science, University of Cape Town, Rondebosch, Cape Town, 7700.

[13] Sarit Kraus, Daniel Lehmann, and Menachem Magidor. 1990. Nonmonotonic reasoning, preferential models and cumulative logics. *Artificial intelligence* 44, 1-2 (1990), 167–207.

[14] Gerhard Lakemeyer and Bernhard Nebel. 2005. Foundations of knowledge representation and reasoning: A guide to this volume. *Foundations of knowledge representation and reasoning* (2005), 1–12.

[15] Daniel Lehmann. 1995. Another perspective on default reasoning. *Annals of Mathematics and Artificial Intelligence* 15, 1 (1995), 61–82. https://doi.org/10.1007/BF01535841

[16] Daniel Lehmann and Menachem Magidor. 1992. What does a conditional knowledge base entail? *Artificial intelligence* 55, 1 (1992), 1–60. https://doi.org/10.1016/0004-3702(92)90041-U

[17] Kody Moodley, Thomas Meyer, and Ivan José Varzinczak. 2012. A defeasible reasoning approach for description logic ontologies. In *Proceedings of the South African Institute for Computer Scientists and Information Technologists Conference.* 69–78.

[18] Yoav Shoham. 1987. A semantic approach to nonmonotonic logics. In *Proceedings of the Tenth International Joint Conference on Artificial Intelligence (IJCAI), 1987.*